Reading 42　Portfolio Risk and Return: Part I

1. Return

LOS A. calculate and interpret major return measures and describe their appropriate uses.
LOS B. describe characteristics of the major asset classes that investors consider in forming portfolios

- 實際報酬率(事後 Ex Post)

  (1) 單期報酬率(Holding-period return, HPR)[1]：Return= $\dfrac{P_t - P_0 + Div + Int}{P_0}$

  此外單期報酬又稱之為期間報酬率(Holding-period return, HPR)，將投資期間所有的投資收益加總(資本利得、利息所得、股利所得)，除以期初所投資的資金，即資產的期初價格(P_0)。

  (2) 多期報酬率：

  - 算術平均報酬率：$\overline{R} = \dfrac{1}{n}\sum_{t=1}^{n} R_t$
  - 幾何平均報酬率：$\overline{R} = [(1+R_1)(1+R_2).....(1+Rn)]^{\frac{1}{n}} - 1$
  - Money-weighted rate of return (like IRR)

  (3) Annualized return(年化報酬率)：

  - 有效年利率(Effective annual interest rate)：指計息方式採複利計算時，本金投入一年實際賺得之利率，又稱間斷複利，複利次數愈高 EAR 愈高。

  ◎ 計算 EAR 可逕以下列公式計算:

  $\text{EAR} = (1+\dfrac{\text{K}}{\text{m}})^m - 1$ 式中 K，m 分別為名目**年**利率及一年計息之次數。

  - 連續複利：如果一年可以複利無限多次，期初(T_0)1 元到期末(T_1)可得e^k，若是經過 n 年，則期末終值為$e^{k \cdot n}$

- 預期報酬率(事前 Ex Ante)：　$\text{E(R)} = \sum_{i=1}^{n} P_i R_i$

【Example】Computation of Returns
Ulli Lohrmann and his wife, Suzanne Lohrmann, are planning for retirement and want to compare the past performance of a few mutual funds they are considering for investment. They believe that a comparison over a five-year period would be

[1] 股票沒有到期日，投資者持有股票可能短則幾天、長則幾年，因此不以 EAR(有效年利率)來衡量投資報酬，而以持有期間報酬率(holding period return)來反映投資者在一定的持有期間內所獲得的股息收入和資本利得占期初投入本金的比率。

appropriate. They are given the following information about the Rhein Valley Superior Fund that they are considering.

Year	Assets Under Management at the Beginning of Year ($)	Net return (%)
1	30 million	15
2	45 million	-5
3	20 million	10
4	25 million	15
5	35 million	3

The Lohrmann are interested in aggregating this information for ease of comparison with other funds.

1. Compute the holding period return for the five-year period.
2. Compute the arithmetic mean annual return.
3. Compute the geometric mean annual return. How does it compare with the arithmetic mean annual return?
4. The Lohrmann want to earn a minimum annual return of 5 percent. Is the money-weighted annual return greater than 5%?

【Ans】

1. HPR=$(1+R_1)(1+R_2)(1+R_3)(1+R_4)(1+R_5)-1=1.15\times0.95\times1.1\times1.15\times1.03-1=42.35\%$
2. (0.15-0.05+0.1+0.15+0.03)/5=7.6%
3. $(1+R_1)(1+R_2)(1+R_3)(1+R_4)(1+R_5)^{(1/5)}-1=1.15\times0.95\times1.1\times1.15\times1.03^{(1/5)}-1=7.32\%$
4. Money-weighted rate of return

$CF_0=-30.00$ $CF_1=-10.50$ $CF_2=22.75$ $CF_3=-3.00$ $CF_4=-6.25$ $CF_5=36.05$

以 5%折現

PV=

$-30+(-10.5)/1.05+22.75/1.05^2+(-3)/1.05^3+(-6.25)/105^4+36.05/1.05^5=1.1471$

【Example】Annualized returns

London Arbitrageurs, PLC employs many analysts who devise and implement trading strategies. Mr. Brown is trying to evaluate three trading strategies that have been used for different periods of time.

- Keith believes that he can predict share price movements based on earnings announcements. In the last **100 days** he has earned a return of **6.2%.**
- Thomas has been very successful in predicting daily movements of the Australian dollar and the Japanese yen based on the carry trade. In the last **4 weeks**, he has earned **2%** after accounting for all transaction costs.
- Lisa follows the fashion industry and luxury retailers. She has been investing in

these companies for the last 3 months. Her return is 5%.
Mr. Brown wants to give a prize to the best performer but is somewhat confused by the returns earned over different periods. Annualized returns in all three cases and advise Mr. Brown.

【Ans】

Keith : $(1+0.062)^{(365/100)}-1=24.55\%$

Thomas : $(1+0.02)^{(52/4)}-1=29.36\%$

Lisa : $(1+0.05)^{4}-1=21.55\%$

【預期報酬】T 股票有下列一年後預期價格之機率分配：

狀態	機率	價格
1	0.25	$50
2	0.40	$60
3	0.35	$70

若今日你（妳）買 T 股票$55，而來年 T 股票會配現金股利$4，你（妳）持有 T 股之預期持有期間報酬(Holding-period Return)為何？

(A)17.72% (B)18.18% (C)17.91% (D)18.89%

【答】(B)

E(P)=0.25×50+0.4×60+0.35×70=61，HPR=((61-55)+4)/55=18.18%

2. Risk (Variance and standard deviation)

LOS C. calculate and interpret the mean, variance, and covariance (or correlation) of asset returns based on historical data.
LOS E. calculate and interpret portfolio standard deviation.

指資產價格或報酬率的波動幅度，波動幅度愈大，則風險愈大；反之，波動幅度愈小，則風險愈小。

➢ 單一證券的風險：

(1) 變異數：$V(R)=E[R_i-E(R_i)]^2=\sum_{t=1}^{n}[R_i-E(R_i)]^2\times P_i=\sigma^2$

(2) 標準差：$\sigma_i=\sqrt{V(R_i)}$

(3) 變異係數(Coefficient of Variation)：指該證券報酬率的標準差除以該證券的期望報酬率，表示獲得一單位預期報酬所承擔的風險，該比率愈大表示承擔的風險愈大。

$$\text{公式}: CV_i=\frac{\sigma_i}{E(R_i)}$$

➢ 多變數的風險分析

(1) 共變異數(Covariance)：

➔衡量兩變數間的相互影響程度

$\sigma_{X,Y}$=COV[X,Y]=E[X-E(X)][Y-E(Y)]=E(XY)-E(X)E(Y)

(2) 相關係數(Correaltion Coefficient)：$\rho_{XY}=\dfrac{\sigma_{X,Y}}{\sigma_X\sigma_Y},-1\le\rho_{XY}\le 1$

註：符號整理

變異數	V(X)	σ_X^2，σ_{XX}
標準差	$\sqrt{V(X)}$	σ_X
共變異數	COV(X,Y)	$\sigma_{X,Y}$

- 報酬率、變異數與共變異數基本運算：
  - (1) E(2X)=2E(X)
  - (2) E(2X+3Y)=2E(X)+3E(Y)
  - (3) V(2X)=2^2V(X)
  - (4) COV(2X,3Y)= 2×3COV(X,Y)
  - (5) COV(X, C)=0，C 為常數
  - (6) V(2X+3Y)= 2^2V(X)+ 3^2V(Y)+2×2×3COV(X,Y)

【Example】Risk

明年景氣	各情況之機率	可口公司	百事公司
成長	30%	65%	20%
持平	40%	15%	15%
衰退	30%	-35%	10%

(1) 可口與百事公司明年預期的報酬率變異數及標準差，分別為多少

(2) 如用運用變異係數來衡量明年度可口公司和百事公司比較值得投資

(3) 若將資金平均分配於可口公司百事公司形成投資組合，其投資組合的變異數為何？

【Ans】

可口公司預期報酬率=15%；百事公司預期報酬率=15%

(1)

可口公司

明年景氣	Prob.	Return	Prob.×[R-E(R)]^2
成長	30%	65%	30%×(65%-15%)^2=7.5%
持平	40%	15%	40%×(15%-15%)^2=0%
衰退	30%	-35%	30%×(-35%-15%)^2=7.5%

$$V(R)=\sum_{t=1}^{n}[R_i-E(R_i)]^2\times P_i=15\%$$

$$\sigma=\sqrt{V(R)}=0.387$$

百事公司

明年景氣	Prob.	Return	Prob.×[R-E(R)]^2
成長	30%	20%	30%×(20%-15%)^2=0.075%
持平	40%	15%	40%×(15%-15%)^2=0%
衰退	30%	10%	30%×(10%-15%)^2=0.075%

$$V(R)=\sum_{t=1}^{n}[R_i-E(R_i)]^2\times P_i=0.15\%$$

$$\sigma=\sqrt{V(R)}=0.0387$$

(2)

公司	預期報酬率	標準差	變異係數
可口公司	15%	0.387	0.387/15%=2.58
百事公司	15%	0.0387	0.0387/15%=0.258

百事公司較值得投資，因為每一單位的預期報酬所對應的風險(標準差)較小

(3)

成長：Rp=65%×1/2+20%×1/2=42.5%

持平：Rp=15%×1/2+15%×1/2=15%

衰退：Rp=-35%×1/2+10%×1/2=-12.5%

E(Rp)=30%×42.5%+40%×15%+30%×(-12.5%)=15%

V(Rp)=30%×(42.5%-15%)^2+40%×(15%-15%)^2+30%×(-12.5%-15%)^2=0.0454

3. Two-assets Portfolio

LOS D. explain risk aversion and its implications for portfolio selection.
LOS F. describe the effect on a portfolio's risk of investing in assets that are less than perfectly correlated.

➢ **基本原理**

證券	投資權重	預期報酬率	標準差
A	W_A	0.10	0.08
B	W_B	0.22	0.20

其中 A 與 B 是風險性資產，某人將資金分成 W_A 和 W_B 分別投入 A 證券和 B 證券(易言之 $W_A+W_B=1$)，所形成的兩資產投資組合其預期報酬率和投資組合標準差如下：

(1) 預期報酬率：$E(R_p)=W_AE(R_A)+W_BE(R_B)$

$= W_AE(R_A)+(1-W_A)E(R_B)$…………..………(1)

(2) 投資組合變異數：

$$\sigma_P^2 = W_A{}^2 \cdot \sigma_A^2 + W_B{}^2\sigma_B^2 + 2W_AW_B\sigma_{A,B} = W_A{}^2 \cdot \sigma_A^2 + W_B{}^2\sigma_B^2 + 2W_AW_B\rho_{A,B}\sigma_A\sigma_B$$

$$= W_A{}^2 \cdot \sigma_A^2 + (1-W_A)^2\sigma_B^2 + 2W_A(1-W_A)\rho_{A,B}\sigma_A\sigma_B \text{ ....(2)}$$

(3) 投資組合標準差：$\sigma_p = \sqrt{\sigma_p^2}$

【Example】Return and Risk of a Two-asset Portfolio

Assume that as a US investor, you decide to hold a portfolio with 80% invested in the S&P 500 US stock index and the remaining 20% in the MSCI Emerging Markets index. The expected return is 9.93% for the S&P 500 and 18.2% for the Emerging Market index. The risk (standard deviation) is 16.21% for the S&P 500 and 33.11% for the Emerging Market index. What will be **<u>the portfolio's expected return and risk</u>** given that the covariance between the S&P 500 and the Emerging Market index is 0.5 %?

【Ans】

$\sigma_p^2 =$

$(0.8^2 \times 0.1621^2) + (0.2^2 \times 0.3311^2) + (2 \times 0.80 \times 0.20 \times 0.005) = 0.02281$

σ_p=0.15103=15.10%

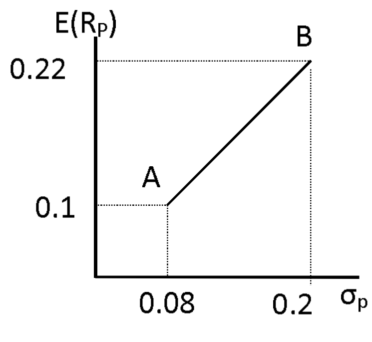
- **<u>兩資產投資組合軌跡線：</u>**

證券	投資權重	預期報酬率	標準差
A	W_A	0.10	0.08
B	W_B	0.22	0.20

相關係數的情形	投資組合軌跡線

完全正相關：當 $\rho_{A,B}$ =+1.0 時，將上表中 A、B 證券的資料帶入公式 4.1 及 4.2 可求出下表結果，並繪圖如左：

W_A	1	0.8	0.5	0.2	0
$E(R_P)$	0.1	0.124	0.16	0.196	0.22
σ_P^2	0.0064	0.010816	0.0196	0.03098	0.04
σ_P	0.08	0.104	0.14	0.176	0.2



<table>
<tr><th>相關係數的情形</th><th>投資組合軌跡線</th></tr>
<tr><td>
完全負相關：當 $\rho_{A,B}$=－1.0 時，將上表中 A、B 證券的資料帶入公式 4.1 及 4.2 可求出下表結果，並繪圖如左：
<table>
<tr><td>W_A</td><td>1</td><td>0.8</td><td>0.7143</td><td>0.5</td><td>0.2</td></tr>
<tr><td>$E(R_P)$</td><td>0.1</td><td>0.124</td><td>0.134</td><td>0.16</td><td>0.196</td></tr>
<tr><td>σ_P^2</td><td>0.0064</td><td>0.00058</td><td>0</td><td>0.0036</td><td>0.0207</td></tr>
<tr><td>σ_P</td><td>0.08</td><td>0.024</td><td>0</td><td>0.06</td><td>0.144</td></tr>
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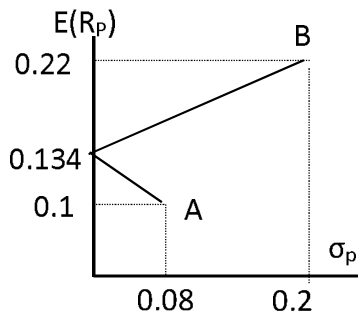

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<tr><td>
無相關：當 $\rho_{A,B}$=0 時，可求出下表結果
<table>
<tr><td>W_A</td><td>1</td><td>0.862</td><td>0.8</td><td>0.5</td><td>0.2</td></tr>
<tr><td>$E(R_P)$</td><td>0.1</td><td>0.117</td><td>0.124</td><td>0.16</td><td>0.196</td></tr>
<tr><td>σ_P^2</td><td>0.006</td><td>0.006</td><td>0.006</td><td>0.012</td><td>0.021</td></tr>
<tr><td>σ_P</td><td>0.08</td><td>0.074</td><td>0.075</td><td>0.108</td><td>0.144</td></tr>
</table>
【註】：若 A 與 B 統計上獨立，則 A 與 B 的相關係數為 0，但相關係數為 0 不一定表示 A 與 B 獨立。(P(A∩B)=P(A)P(B)即為獨立)
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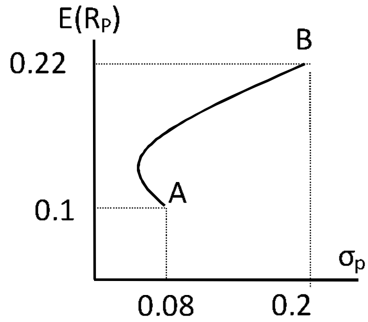

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【相關係數】假設兩種股票的標準差不同，則在不允許賣空(short sale)的情況下，兩種股票之間的相關係數為何？才能組合成一個無風險的投資組合：(A)+1.00 (B)0.00 (C)-1.00 (D)0.5

【答】(C)

【Example】Portfolio risk as correlation varies

Consider two risky assets that have returns variances of 0.0625 and 0.0324, respectively. The assets' standard deviations of returns are then 25% and 18%, respectively. Calculate the variances and standard deviations of portfolio returns for an equal-weighted portfolio of the tow assets when their correlation of returns is 1, 0.5, and -0.5.

【Ans】

1. ρ=+1 $\sigma^2=0.215^2=0.046225$ σ=21.5%
2. ρ=+0.5 $\sigma^2=0.5^2\times0.0625+.5^2\times0.0324+2\times0.5\times0.5\times0.25\times0.18=0.034975$
   σ=18.7%
3. ρ=+0 $\sigma^2=0.5^2\times0.0625+.5^2\times0.0324=0.023725$
   σ=15.4%
4. ρ=-0.5 $\sigma^2=0.5^2\times0.0625+.5^2\times0.0324+2\times0.5\times0.5\times(-0.5)\times0.25\times0.18=0.012475$
   σ=11.17%